NAG Toolbox for MATLAB

c02an

1 Purpose

c02an determines the roots of a quartic equation with complex coefficients.

2 Syntax

```
[zeror, zeroi, errest, ifail] = c02an(e, a, b, c, d)
```

3 Description

c02an attempts to find the roots of the quartic equation

$$ez^4 + az^3 + bz^2 + cz + d = 0,$$

where e, a, b, c and d are complex coefficients with $e \neq 0$. The roots are located by finding the eigenvalues of the associated 4 by 4 (upper Hessenberg) companion matrix H given by

$$H = \begin{pmatrix} 0 & 0 & 0 & -d/e \\ 1 & 0 & 0 & -c/e \\ 0 & 1 & 0 & -b/e \\ 0 & 0 & 1 & -a/e \end{pmatrix}.$$

The eigenvalues are obtained by a call to f08ps. Further details can be found in Section 8.

To obtain the roots of a cubic equation, c02am can be used.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **e – complex scalar**

e, the coefficient of z^4.

Constraint: **e** $\neq (0.0, 0.0)$.

2: **a – complex scalar**

a, the coefficient of z^3.

3: **b – complex scalar**

b, the coefficient of z^2.

4: **c – complex scalar**

c, the coefficient of z.

5: **d – complex scalar**

d, the constant coefficient.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **zeror**(4) **– double array**

2: **zeroi**(4) **– double array**

zeror(i) and **zeroi**(i) contain the real and imaginary parts, respectively, of the ith root.

3: **errest**(4) **– double array**

errest(i) contains an approximate error estimate for the ith root.

4: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{e} = (0.0, 0.0)$.

ifail $= 2$

The companion matrix H cannot be formed without overflow.

ifail $= 3$

The iterative procedure used to determine the eigenvalues has failed to converge.

7 Accuracy

If **ifail** $= 0$ on exit, then the ith computed root should have approximately $|\log_{10}(\mathbf{errest}(i))|$ correct significant digits.

8 Further Comments

The method used by the function consists of the following steps, which are performed by routines from LAPACK in Chapter F08.

(a) Form matrix H.

(b) Apply a diagonal similarity transformation to H (to give H').

(c) Calculate the eigenvalues and Schur factorization of H'.

(d) Calculate the left and right eigenvectors of H'.

(e) Estimate reciprocal condition numbers for all the eigenvalues of H'.

(f) Calculate approximate error estimates for all the eigenvalues of H' (using the 1-norm).

9 Example

```
e = complex(1, 0);
a = complex(-10, 2);
b = complex(48, -10);
c = complex(-100, 28);
d = complex(96, 0);
[zeror, zeroi, errest, ifail] = c02an(e, a, b, c, d)
```

```
zeror =
    3.0000
    1.0000
    2.0000
    4.0000
zeroi =
    3.0000
    1.0000
   -2.0000
   -4.0000
errest =
   1.0e-13 *
    0.1550
    0.1178
    0.2142
    0.2018
ifail =
           0
```